Phenomenological impact of the resummation of logs of α in heavy quarkonium

Antonio Pineda[a] [*]

[a]Dept. d'Estructura i Constituents de la Matèria, U. Barcelona, Diagonal 647, E-08028 Barcelona, Spain

Here we would like to review recent progress on the resummation on logarithms of α_s in heavy quarkonium. We will mainly focus on the phenomenological relevance of these achievements. Determinations of the $\eta_b(1S)$ mass, $B_c(1S)$ hyperfine splitting, inclusive electromagnetic decays and implications for $t\text{-}\bar{t}$ production near threshold.

1. Introduction

Heavy quark-antiquark systems near threshold are characterized by the small relative velocity v of the heavy quarks in their center of mass frame. This small parameter produces a hierarchy of widely separated scales: m (hard), mv (soft), mv^2 (ultrasoft), ... The factorization between them is efficiently achieved by using effective field theories, where one can organize the calculation as various perturbative expansions on the ratio of the different scales effectively producing an expansion in v. The terms in these series get multiplied by parametrically large logs: $\ln v$, which can also be understood as the ratio of the different scales appearing in the physical system. Again, effective field theories are very efficient in the resummation of these large logs once a non-relativistic renormalization group (NRG) analysis of them has been performed. We will review in this paper recent progress on the resummation of the above logarithms, within the context of pN-RQCD [1], in the weak coupling regime ($v \sim \alpha_s$). Besides the pure theoretical interest of these computations, they may also have an important phenomenological impact in several situations. Let us enumerate a few of them. The determination of the bottom and charm masses (using the experimental value of the ground state heavy quarkonium masses or non-relativistis sum rules). The determination of the $\eta_b(1S)$ mass, the hyperfine

splitting (HFS) of the ground state B_c system, or theoretical improved determinations of the η_c. One can also try to obtain improved determinations for the inclusive electromagnetic decays of the heavy quarkonium. On the other hand the application of this program to $t\text{-}\bar{t}$ production near threshold at the Next Linear Collider is one of the main motivations to undergo these computations. In this paper we review the phenomenological analysis already available in the literature and outline possible future work.

2. Hyperfine splitting

Analytical expressions for the HFS of heavy quarkonium (for the equal and non-equal mass case) are available with leading log (LL) [2,3] and next-to-leading log (NLL) [4,5] accuracy.

For the case of bottomonium, these results have been used in Ref. [4] to give predictions for the mass of the $\eta_b(1S)$ (using the experimentally very well known value of $M_{\Upsilon(1S)}$) with great precision

$$M(\eta_b(1S)) = 9421 \pm 11\,(\text{th})\,{}^{+9}_{-8}\,(\delta\alpha_s)\ \text{MeV}, \quad (1)$$

where the errors due to the high-order perturbative corrections and the nonperturbative effects are added up in quadrature in "th", whereas "$\delta\alpha_s$" stands for the uncertainty in $\alpha_s(M_Z) = 0.118 \pm 0.003$. This prediction is of great experimental interest. The discovery of the η_b meson is one of the primary goals of the CLEO-c research program [6] and there are experimental proposals for its detection at Tevatron too [7]. It has also been argued that the HFS can be used to search

for new physics [8]. Therefore, an accurate prediction of its mass $M(\eta_b)$ is thus a big challenge and a test for the QCD theory of heavy quarkonium. For instance, this prediction can be compared with those obtained either in lattice [9], potential models (see for instance Ref. [10]) or sum rules [11]. It seems to be a general trend that our result is larger than the lattice predictions and smaller than most of the potential model results. We would also like to remark that the inclusion of resummation of logarithms has a sizable effect in the determination of the $\eta_b(1S)$ mass. We illustrate this fact in Fig. 1 from Ref. [4]. In this figure, the HFS for the bottomonium ground state is plotted as a function of μ in the LO, NLO, LL, and NLL approximations. As we see, the LL curve shows a weaker scale dependence compared to the LO one. The scale dependence of the NLO and NLL expressions is further reduced, and, moreover, the NLL approximation remains stable up to smaller scales than the fixed-order calculation. At the scale $\mu' \approx 1.3$ GeV, which is close to the inverse Bohr radius, the NLL correction vanishes. Furthermore, at $\mu'' \approx 1.5$ GeV, where $\alpha_s^{LL} = 0.319$, the result becomes independent of μ; $i.e.$, the NLL curve shows a local maximum. This suggests a nice convergence of the logarithmic expansion despite the presence of the ultrasoft contribution with α_s normalized at the rather low scale $\bar{\mu}^2/m_b \sim 0.8$ GeV. By taking the difference of the NLL and LL results at the local maxima as a conservative estimate of the error due to uncalculated perturbative higher-order contributions, one gets $E_{\text{hfs}} = 39 \pm 8$ MeV. A similar error estimate is obtained by the variation of the normalization scale in the physically motivated soft region $1 - 3$ GeV.

For the case of charmonium, the use of perturbation theory is more doubtful, even though there has been some attempts in this direction recently [12] (see also [13] for the B_c), yet one can try and see what comes out. This is specially important since even unquenched attempts to obtain the HFS of charmonium from lattice undershot the experimental value by around 20% [14]. The results obtained in Ref. [4] are given in Fig. 2 along with the experimental value 117.7 ± 1.3 MeV [15]. The local maximum of the NLL curve corre-
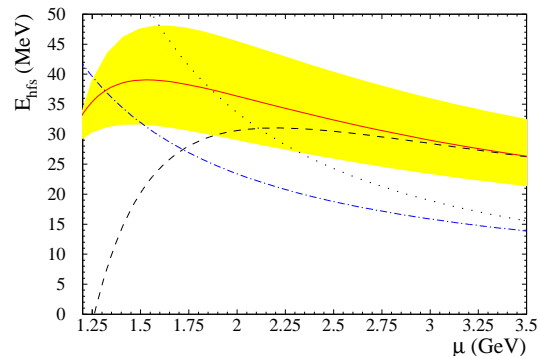

Figure 1. *HFS of 1S bottomonium as a function of the renormalization scale μ in the LO (dotted line), NLO (dashed line), LL (dot-dashed line), and NLL (solid line) approximations. For the NLL result, the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$.*

sponds to $E_{\text{hfs}} = 104$ MeV and $\alpha_s^{LL} = 0.534$. We should emphasize the crucial role of the resummation to bring the perturbative prediction closer to the experimental figure. Therefore, in Ref. [4] the whole difference of ≈ 14 MeV between the perturbative prediction and the experimental value for the HFS of the ground state of charmonium was used to estimate the size of the non-perturbative effects. In any case, within the power counting assumed in Ref. [4], these non-perturbative effects are beyond the accuracy of this computation and are added to the errors. Taking into account that they are suppressed by the inverse heavy-quark mass as $1/(\alpha_s m_q)^2$ due to the multipole expansion, one obtains ≈ 3.5 MeV for the typical size of the nonperturbative contribution to the HFS in bottomonium. For the estimate of the nonperturbative error, this number was multiplied by two, which was used above for the determination of the theoretical error. These formulae has also been applied to $n = 2$ excited states. For bottomonium, one obtains $E_{\text{hfs}}(2S)/E_{\text{hfs}}(1S) = 0.25$. For charmonium, our perturbative estimate $E_{\text{hfs}}(2S)/E_{\text{hfs}}(1S) = 0.37$ also reasonably agrees
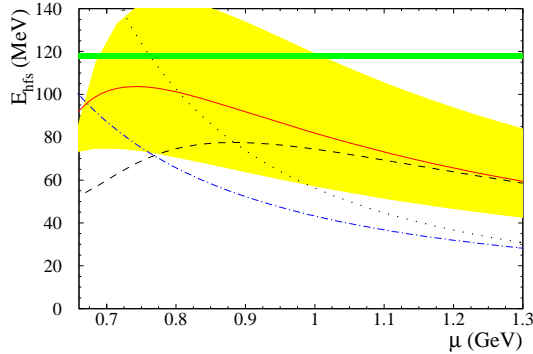
Figure 2. *HFS of 1S charmonium as a function of the renormalization scale μ in the LO (dotted line), NLO (dashed line), LL (dot-dashed line), and NLL (solid line) approximations. For the NLL result, the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$. The horizontal band gives the experimental value 117.7 ± 1.3 MeV [15].*

with the result 0.41 ± 0.03 of the recent experimental measurements [16]. Although one cannot rely on the (even NRG-improved) perturbative analysis of the excited charmonium states, the above agreement suggests that the nonperturbative effects are small, at least for the ground state.

It has also been possible to give a good prediction for the HFS of the $B_c(1S)$ system[2] [5]

$$M(B_c^*) - M(B_c) = 65 \pm 24 \,(\text{th}) \,^{+19}_{-16} \,(\delta\alpha_s)\,\text{MeV} \quad (2)$$

Potential models appear to obtain slightly larger numbers [17]. As in the previous cases the inclusion of logarithms appears to be a large effect. In Fig. 3 from ref. [5], the HFS for the charm-bottom quarkonium ground state is plotted as a function of ν in the LO, NLO, LL, and

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[2]In this case it is the pseudoscalar, $B_c(1S)$, which has been measured, nevertheless not with enough precision to allow a good absolute determination of the $B_c^*(1S)$ mass. However, with improved statistics and precision of the B_c data, this result can be considered as a prediction for the B_c^* meson mass.

NLL approximations for the hard matching scale value $\nu_h = 1.95$ GeV. As we see, the LL curve shows a weaker scale dependence compared to the LO one. The scale dependence of the NLO and NLL expressions is further reduced, and, moreover, the NLL approximation remains stable at the physically motivated scale of the inverse Bohr radius, $C_F\alpha_s m_r \sim 0.9$ GeV, where the fixed-order expansion breaks down. At the scale $\nu' \approx 0.85$ GeV, which is close to the inverse Bohr radius, the NLL correction vanishes. Furthermore, at $\nu'' = 0.92$ GeV, the result becomes independent of ν; i.e., the NLL curve shows a local maximum corresponding to $E_{\text{hfs}} = 65$ MeV, which is taken as the central value of the estimate. The NLL curve also shows an impressive stability with respect to the hard matching scale variation in the physical range $m_c < \nu_h < m_b$ and has a local maximum at $\nu_h = 1.95$ GeV, which is taken for the numerical estimates. All this suggests a nice convergence of the logarithmic expansion despite the presence of the ultrasoft contribution where α_s is normalized at the rather low scale $\bar{\nu}^2/\nu_h \sim 0.5$ GeV. Let us discuss the accuracy of our result. For a first estimate of the error due to uncalculated higher-order contributions, we take 9 MeV, the difference of the NLL and LL results at the local maxima. A different estimate can be obtained by varying the normalization scale in the physical range $0.8 \leq \nu \leq 1.4$ GeV. In this case the difference with the maximum is 16 MeV. Being conservative, we take this second number for our estimate of the perturbative error. Within the power counting assumed in Ref. [5], the nonperturbative effects are beyond the accuracy of the computation and added to the errors. They are also inferred using charmonium data in the same way than above and ≈ 18 MeV for the error due to the nonperturbative contribution to the HFS in B_c was obtained.

3. Production and Annihilation rates

There has also been much progress on the resummation of large logarithms appearing in the annihilation of a heavy quarkonium into leptons, photons or light hadrons, as well as its production in e^+e^- or $\gamma\gamma$ collisions.
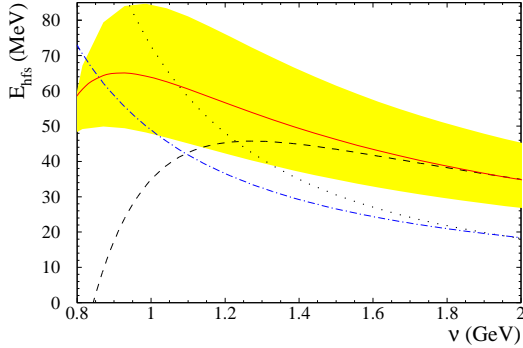
Figure 3. *HFS for charm-bottom quarkonium as the function of the renormalization scale ν in LO (dotted line), NLO (dashed line), LL (dot-dashed line), and NLL (solid line) approximation for $\nu_h = 1.95$ GeV. For the NLL result the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$.*

Figure 4. *The spin ratio as the function of the renormalization scale ν in LO$\equiv$LL (dotted line), NLO (short-dashed line), NNLO (long-dashed line), NLL (dot-dashed line), and NNLL (solid line) approximation for the (would be) toponium ground state with $\nu_h = m_t$. For the NNLL result the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$. Note that for the vertical axis the zero is suppressed.*

The resummation of the large logarithms of the heavy quark velocity to all orders in α_s has been advocated as a tool to improve the behaviour of the perturbative expansion for $t\bar{t}$ threshold production [18]. Currently, the complete next-to-leading logarithmic (NLL) approximation for the production and annihilation rates is available [19,20]. In Ref. [21] a phenomenological analysis of the NLL result for the electromagnetic inclusive decays of the heavy quarkonium was made. The first attempt to go beyond the NLL approximation [18] suggested a very good convergence of the logarithmic expansion. In particular, an accuracy of 2-3% was claimed for the cross section of $t\bar{t}$ threshold production. However, subsequent calculations of some next-to-next-to-leading logarithmic (NNLL) terms [22], which had not been taken into account in Ref. [18], casted serious doubts on this estimate. Thus, the full calculation of the NNLL corrections, which still remains elusive, is unavoidable to draw definite conclusions. In Ref. [23], it has recently been derived
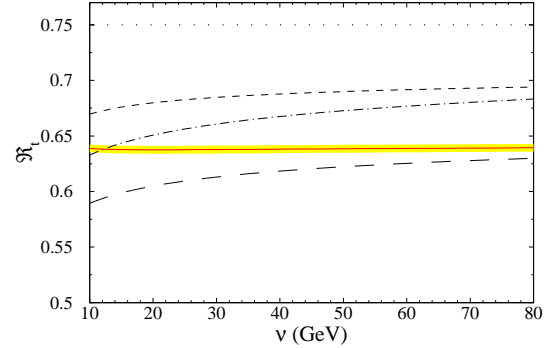
the *complete* NNLL result for the spin dependent part of the heavy quarkonium production annihilation rates, which includes the terms of the form $\alpha_s^{n+2} \ln^n \alpha_s$ for all n and applied to the heavy quarkonium phenomenology. In Figs. 4, 5 and 6, the spin ratio is plotted as a function of ν in the various logarithmic and fixed-order approximations for the (hypothetical) toponium, bottomonium and charmonium ground states, respectively. As we see, in the second order the convergence and stability of the result with respect to the scale variation is substantially improved if one switches from the fixed-order to the logarithmic expansion. We want to remark that the ν dependence of the NLL approximation is slightly worse than at NLO. This is due to the artificially small ν dependence at NLO which is likely due to the fact that at this order only the hard scale enters.
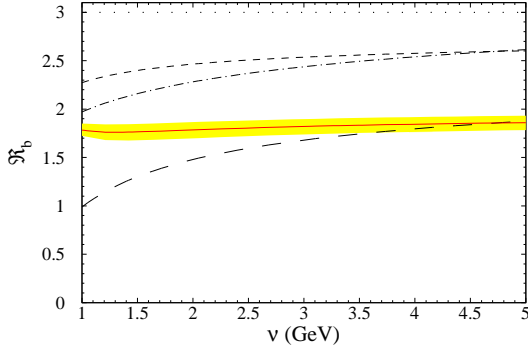
Figure 5. *The spin ratio as the function of the renormalization scale ν in LO≡LL (dotted line), NLO (short-dashed line), NNLO (long-dashed line), NLL (dot-dashed line), and NNLL (solid line) approximation for the bottomonium ground state with $\nu_h = m_b$. For the NNLL result the band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$.*
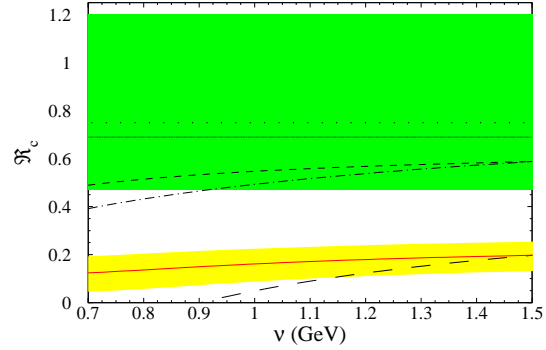

Figure 6. *The spin ratio as the function of the renormalization scale ν in LO≡LL (dotted line), NLO (short-dashed line), NNLO (long-dashed line), NLL (dot-dashed line), and NNLL (solid line) approximation for the charmonium ground state with $\nu_h = m_c$. For the NNLL result the lower (yellow) band reflects the errors due to $\alpha_s(M_Z) = 0.118 \pm 0.003$. The upper (green) band represents the experimental error of the ratio [15] where the central value is given by the horizontal solid line.*

Let us first consider the top quark case. In particular, the ratio of the cross sections of the resonance $e^+e^- \to t\bar{t}$ and $\gamma\gamma \to t\bar{t}$ production. As one can see in Fig. 4, the logarithmic expansion shows perfect convergence and the NNLL correction vanishes at the scale $\nu \approx 13$ GeV, which is close to the physically motivated scale of the inverse Bohr radius $\alpha_s m_t/2$. For illustration, at the scale of minimal sensitivity, $\nu = 20.2$, GeV we have

$$\frac{\sigma_{\mathrm{res}}(e^+e^- \to t\bar{t})}{\sigma_{\mathrm{res}}(\gamma\gamma \to t\bar{t})} = \frac{1}{3Q_t^2} \left(1 - 0.132 - 0.018\right) . \quad (3)$$

However, it is not clear if the nice behaviour of the logarithmic expansion also holds for the spin-independent part of the threshold cross section. A possible problem is connected to the ultra-soft contribution, which is enhanced by the larger value of α_s at the ultrasoft scale. Whereas it is suppressed in the spin ratio by the fifth power of α_s, for the spin-independent part it already

contributes at $\mathcal{O}(\alpha_s^3)$ and can destabilize the expansion.

For bottomonium, the logarithmic expansion shows nice convergence and stability (c.f. Fig. 5) despite the presence of ultrasoft contributions with α_s normalized at a rather low scale ν^2/m_b. At the same time, the perturbative corrections are important and reduce the leading order result by approximately 41%. For illustration, at the scale of minimal sensitivity, $\nu = 1.295$ GeV, we have the following series:

$$\frac{\Gamma(\Upsilon(1S) \to e^+e^-)}{\Gamma(\eta_b(1S) \to \gamma\gamma)} = \frac{1}{3Q_b^2} \left(1 - 0.302 - 0.111\right) . (4)$$

In contrast, the fixed-order expansion blows up at the scale of the inverse Bohr radius.

So far we have discussed the perturbative corrections to the Coulomb-like quarkonium. How-

ever, in contrast to the $t\bar{t}$ system, for bottomonium nonperturbative contributions can be important. In our case the interaction of the quark-antiquark pair to the nonperturbative gluonic field is suppressed by v through the multipole expansion in the same way than for the HFS computation. In this case however they only contribute in the N^4LL approximation, far beyond the precision of this computation. Note that the nonperturbative contribution to the decay rates ratio is suppressed by a factor v^2 in comparison to the binding energy and decay rates, where the leading nonperturbative effect is due to chromo-electric dipole interaction. Thus, by using the available experimental data on the Υ meson as input, one can predict the production and annihilation rates of the yet undiscovered η_b meson. In particular, one can predict the $\eta_b(1S)$ decay rate using the experimental value for the $\Upsilon(1S)$ decay rate and the following figure was obtained in Ref. [23]

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.659 \pm 0.089(\text{th.})^{+0.019}_{-0.018}(\alpha_\text{s}) \\ \pm 0.015(\text{exp.})\,\text{keV}, \qquad (5)$$

where $\nu = 1.295$ GeV, the scale of minimal sensitivity, was taken for the central value, the difference between the NLL and NNLL result for the theoretical error and $\alpha_\text{s}(M_Z) = 0.118 \pm 0.003$. The last error in Eq. (5) reflects the experimental error of $\Gamma(\Upsilon(1S) \to e^+e^-) = 1.314 \pm 0.029$ keV [15]. This value considerably exceeds the result for the absolute value of the decay width obtained in Ref. [21] on the basis of the full NLL analysis including the spin-independent part (see Fig. 7):

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.35 \pm 0.1(\text{th.}) \pm 0.05(\alpha_\text{s})\text{KeV.} \quad (6)$$

This can be a signal of slow convergence of the logarithmic expansion for the spin-independent contribution which is more sensitive to the dynamics of the bound state and in particular to the ultrasoft contribution as it has been discussed above. On the other hand, the renormalon effects [24] could produce some systematic errors in the pure perturbative evaluations of the production/annihilation rates. The problem is expected to be more severe for the charmonium case discussed below.
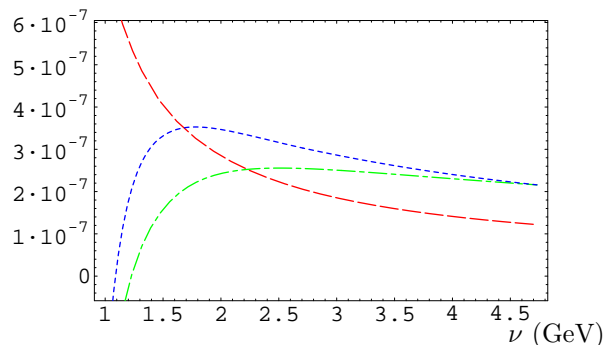

Figure 7. *Plot of $\Gamma(\eta_b(1S) \to \gamma\gamma)$ with LO/LL (dashed line), NLO (dot-dashed line) and NLL (dotted line) accuracy versus the renormalization scale ν.*

We would also like to remark that the one-loop result for $\nu = m_b$ overshoots the NNLL one by approximately 30%. This casts some doubts on the accuracy of the existing α_s determination from the $\Gamma(\Upsilon \to \text{light hadrons})/\Gamma(\Upsilon \to e^+e^-)$ decay rates ratio, which gives $\alpha_s(m_b) = 0.177 \pm 0.01$, well below the "world average" value [15]. The theoretical uncertainty in the analysis is estimated through the scale dependence of the one-loop result. Our analysis of the photon mediated annihilation rates indicates that the actual magnitude of the higher order corrections is most likely quite beyond such an estimate and the theoretical uncertainty given in [15] should be increased by a factor of two. This brings the result for α_s into 1σ distance from the "world average" value.

For the charmonium, the NNLO approximation becomes negative at an intermediate scale between $\alpha_s m_c$ and m_c (c.f. Fig. 6) and the use of the NRG is mandatory to get a sensible perturbative approximation. The NNLL approximation has good stability against the scale variation but the logarithmic expansion does not converge well. This is the main factor that limits the theoretical accuracy since the nonperturbative contribution is expected to be under control. For illustration, at the scale of minimal sensitivity, $\nu = 0.645$ GeV,

one obtains [23]

$$\frac{\Gamma(J/\Psi(1S) \to e^+e^-)}{\Gamma(\eta_c(1S) \to \gamma\gamma)} = \frac{1}{3Q_c^2}\,(1 - 0.513 - 0.326)\ .(7)$$

The central value is 2σ below the experimental one. The discrepancy may be explained by the large higher order contributions. This should not be surprising because of the rather large value of α_s at the inverse Bohr radius of charmonium. For the charmonium HFS, however, the logarithmic expansion converges well and the prediction of the NRG is in perfect agreement with the experimental data. Thus one can try to improve the convergence of the series for the production/annihilation rates by accurately taking into account the renormalon-related contributions. One point to note is that with a potential model evaluation of the wave function correction, the sign of the NNLO term is reversed in the charmonium case [25]. At the same time the subtraction of the pole mass renormalon from the perturbative static potential makes explicit that the potential is steeper and closer to lattice and phenomenological potential models [26]. Therefore, the incorporation of higher order effects from the static potential may improve the agreement with experiment. Finally, we can not avoid mention that the NLL evaluation for the decay is able to reproduce the experimental value (see Fig. 8). Therefore, some extra work needs to be done to clarify these issues.

4. Conclusions and outlook

We have seen that the resummation of logarithms appear to have a large phenomenological impact in the heavy quarkonium physics. This is so in top-, bottom- or charmonium physics.

First, the heavy-quarkonium HFS have been studied in the NLL approximation. The use of the NRG extends the range of μ where the perturbative result is stable to the physical scale of the inverse Bohr radius. The resummation of logarithms is found to be crucial to bring the perturbative prediction closer to the experimental figure of the HFS in charmonium (despite *a priori* unsuppressed nonperturbative effects), and to give reliable predictions for the $\eta_b(1S)$ mass and the
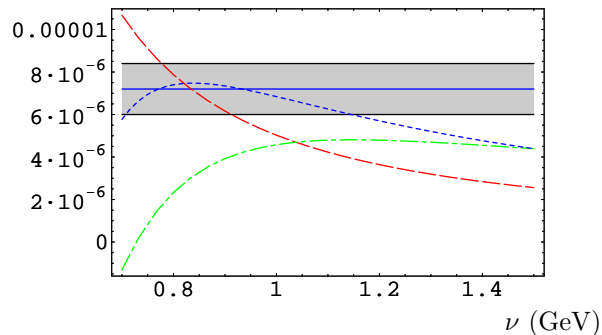

Figure 8. *Plot of $\Gamma(\eta_c(1S) \to \gamma\gamma)$ with LO/LL (dashed line), NLO (dot-dashed line) and NLL (solid line) accuracy versus the renormalization scale ν. The horizontal line and its band give the experimental value and its errors: $\Gamma(\eta_c(1S) \to \gamma\gamma) = 7.2 \pm 1.2\ KeV$ [15].*

$B_c(1S)$ HFS. These results seem to indicate that the properties of the charmonium, B_c and bottomonium ground states are dictated by perturbative dynamics.

In the case of t-$\bar{t}$ production near threshold, the partial NNLL analysis made in Ref. [22] does not seem to show a very nice convergence (even if the absolute value of the corrections is small). Nevertheless, being incomplete, such analysis is scheme dependent. In Ref. [23] a complete result with NNLL accuracy has been obtained for the ratio of the spin one and spin zero production. This is a physical result by itself and therefore scheme independent. In this case a very nice convergence is found. Nevertheless, in this case the contribution due to the ultrasoft scale is suppressed. Therefore, it is premature to draw any definite conclusion for the convergence of the series. We are then eagerly waiting for the complete NNLL evaluation, which, even if difficult, is within reach. This is of utmost importance for the future determinations of the top mass and the Higgs-top coupling at the Next Linear Collider [27].

For the inclusive electromagnetic decays of the bottomonium and charmonium ground states the effects due to the resummation of logarithms ap-

pear to be large and always improve the result compared with the finite order evaluations, yet the errors are large and further work seems to be needed. Here as well, the complete NNLL evaluation would be of utmost importance to further clarify the physical picture. However, one should not forget the possible drawbacks of these determinations. There is an implicit dependence on the ultrasoft scale, which for the case of bottomonium and charmonium is quite low and is related with non-perturbative effects, which should be eventually studied with the help of lattice or models. Moreover, renormalon effects could also play a role.

Further work is ahead. No analysis has been made yet for the non-relativistic sum rules (to obtain them with NNLL accuracy would be a major step to obtain accurate determinations of the bottom and charm masses since they are strongly scale dependent), nor the impact of the resummation of logarithms in the determination of the bottom and charm masses from the ground state masses (which are known with NNLL accuracy [3,20]) estimated.

Acknowledgments

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